UNIVERSITY OF KERALA
Model Question Paper
First Degree Programme in Mathematics
Semester IV
MM 1441 Methods of Algebra and Calculus- II

Time: 3 hours **Maximum Marks: 80**

Section-I

All the first 10 questions are compulsory. They carry 1 mark each.

1. Find $(x^2+x+1)^2$ in $\mathbb{F}_2[x]$.
2. For which values of k in $\mathbb{Q}$, does $x-k$ divide $x^3-kx^2-2x+k+3$?
3. Find the remainder in $\mathbb{Q}[x]$ when $x^{40}-8x^{12}+3$ is divided by x^4-1.
4. If $N(e)$ is the number of elements of U_p which have order e, then, $\sum_{e/p-1} N(e) = \dots\dots$
5. State whether the polynomials $x+2$ and $4x+3$ are associates of each other in $\mathbb{Z}/5\mathbb{Z}[x]$
6. $\lim_{(x,y)\to(0,0)}(x^2+y^2)\ln(x^2+y^2) = \cdots$
7. Find the point at which $f(x,y)=(x-2)^2+(y+1)^2$ has an absolute minimum.
8. Express $\int_0^2\int_0^{\sqrt{x}} f(x,y)dydx$ as an equivalent integral with the order of integration reversed.
9. If $f(x,y)=x^3y^2-5x^2y-2x^5$, find f_{xyy}.
10. Evaluate: $\int_2^4\int_0^1 x^2y\,dxdy$

Section-II

Answer any 8 questions from among the questions 11 to 22.
These questions carry 2 marks each.

11. In $\mathbb{Q}[x]$, when $f(x)$ is divided by $(x^2-3)(x+1)$, the reminder is x^2+2x+5. What is the reminder when $f(x)$ is divided by x^2-3?
12. If R is an integral domain, show that R[x] is also an integral domain.
13. Which of the following polynomials is irreducible in $\mathbb{R}[x]$:
    i. x^2-2 ii. x^2+1 iii. x^2-5x+6 ii. x^3-1
14. Write x^3 in base $x+1$
15. Using Euclid's algorithm find a g.c.d. of x^2-x+4 and x^3+2x^2+3x+2 in $\mathbb{F}_3[x]$
16. Use geometric arguments to evaluate: $\int_0^1\int_0^{\sqrt{1-y^2}} \sqrt{1-x^2-y^2}dxdy$
17. Sketch the domain of $f(x,y)=\ln(1-x-x^2)$
18. Show that $z=e^x\sin y+e^x\cos y$ satisfies Laplace's equation.
19. Suppose that $w=x^3y^2z^4$; $x=t^2$, $y=t+2$, $z=2t^4$. Find the rate of change of w with respect to t at $t=1$ by using the chain rule and check the answer by expressing w as a function of t and differentiating.
20. Locate all relative maxima, relative minima and saddle points, if any, of the function $f(x,y)=x^2+xy+y^2-3x$

21. Find an equation of the tangent plane to the parametric surface: $x = u, y = v, z = u^2 + v^2$

22. Evaluate $\int_0^4 \int_{\sqrt{y}}^2 e^{x^3} dxdy$ by reversing the order of integration

Section-III

Answer any 6 questions from among the questions 23 to 31.

These questions carry 4 marks each.

23. Find a solution of $y^4 = 25y + 156$ by Ferrari's method.

24. Find a solution of $x^3 + 3x = 5$ by Cardano's method.

25. Use Newton's method to approximate the real solution of $x^3 + x - 1 = 0$.

26. For any n, prove that: $\sum_{d/n} \varphi(d) = n$

27. If p is irreducible and f is any polynomial which is not divisible by p, show that the greatest common divisor of p and f is 1.

28. Let $f(x,y) = \frac{x^2}{x^2+y^2}$. Is it possible to define $f(0,0)$ so that f will be continuous at $(0,0)$? Justify your answer.

29. Let $f(x,y) = (x^2 + y^2)^{2/3}$. Show that $f_x(x,y) = \begin{cases} \frac{4x}{3(x^2+y^2)^{1/3}}; & (x,y) \neq (0,0) \\ 0; & (x,y) = (0,0) \end{cases}$

30. Use a double integral in polar coordinates to find the area of the region inside the circle $r = 4\sin\theta$ and outside the circle $r = 2$.

31. Evaluate: $\int_0^2 \int_0^{\sqrt{4-x^2}} \int_{-5+x^2+y^2}^{3-x^2-y^2} xdzdydx$

Section-IV

Answer any 2 questions from among the questions 32 to 35.

These questions carry 15 marks each.

32. State and prove the Division Theorem for $\mathbb{F}[x]$ where $\mathbb{F}$ is a field. Deduce the Reminder Theorem.

33. (a) Prove that any polynomial of degree greater than or equal to 1 in $\mathbb{F}[x]$ where $\mathbb{F}$ is a field is irreducible or factors into a product of irreducible polynomials

    (b) Factor $x^5 - x$ into irreducible polynomials in $\mathbb{Z}/5\mathbb{Z}[x]$.

34. (a) Find the absolute extrema of the function $f(x,y) = x^2 - 3y^2 - 2x + 6y$ on the closed and bounded set R where R is the square with vertices (0,0), (0,2), (2,2) and (2,0).

    (b) Use Lagrange Multiplier Method to find the points on the circle $x^2 + y^2 = 45$ that are closest to and farthest from (1,2).

35. (a) Use double integration to find the volume of the solid bounded by the cylinder $x^2 + y^2 = 9$ and the planes: $z = 0$, $z = 3 - x$.

    (b) Find the surface area of the sphere $x^2 + y^2 + z^2 = 16$ between the planes $z = 1$ and $z = 2$.
