UNIVERSITY OF KERALA
Model Question Paper
First Degree Programme in Physics
Semester IV
MM 1431.1 Mathematics – IV
(Complex Analysis, Fourier Series, Fourier Transforms)

Time: 3 hours **Maximum Marks: 80**

Section-I

All the first 10 questions are compulsory. They carry 1 mark each.

1. Find all roots of the equation $\log z = i\pi/2$
2. Write down the principal value of $\log(-1)$
3. State whether $f(z) = \bar{z}$ is analytic or not.
4. Find the singular points of the function $f(z) = \frac{2z+1}{z(z^2-1)}$
5. Write down the Taylor series for $f(z) = \frac{1}{e^{-z}}$ at $z = 0$
6. Evaluate $\int_C z^2 dz$ where C is any curve joining 0 to $1 + i$
7. Find the residue of $f(z) = \frac{z^2-1}{z^2+z}$ at $z = 0$
8. Write down the Euler formulae for calculating the Fourier coefficients.
9. What is the standard form of Fourier series for an even function?
10. What are the sufficient conditions for the existence of Fourier transform.

Section-II

Answer any 8 questions from among the questions 11 to 22.
These questions carry 2 marks each.

11. Prove that the real and imaginary parts of an analytic function are harmonic.
12. Show that an analytic function is constant if its modulus is constant.
13. Find an analytic function whose real part is $u = e^x(x\cos y - y\sin y)$.
14. Evaluate $\int_C (y - x - 3x^2 i)\, dz$ where $z = x + iy$ and C is the straight line joining 0 to $1 + i$
15. State Cauchy's integral formula. Hence evaluate $\int_C \frac{z^3 dz}{z-2}$ where C is the circle $|z| = 3$
16. Expand $f(z) = \frac{z}{(z-1)\,(z-3)}$ in Taylor series about $z = 1$
17. Expand $f(z) = \frac{z-1}{z^3}$ in positive and negative powers of $z - 1$
18. Find the nature of the singularity of the function $f(z) = \sin\left(\frac{1}{z-1}\right)$
19. Find the residue of $f(z) = \frac{1}{(z-1)^3}$ at its poles.

20. Find the Fourier series of $f(x) = x;\ \ 0 < x < 2\pi$
21. Find the Fourier series of $f(x) = |x|;\ -2 < x < 2$
22. Derive the Fourier transform of $f'(x)$, the derivative of $f(x)$

Section-III
Answer any 6 questions from among the questions 23 to 31.
These questions carry 4 marks each.

23. Show that the function $f(z) = \begin{cases} \frac{(\bar{z})^2}{z}; & z \neq 0 \\ 0; & z = 0 \end{cases}$ is not differentiable at $z = 0$ even though Cauchy-Riemann equations are satisfied there.
24. If a function is analytic, show that it is independent of $\bar{z}$
25. Write different types of isolated singularities. Give one example each.
26. State Cauchy's residue theorem. Hence evaluate $\int_C \frac{e^z\, dz}{\sin z}$ where C is the circle $|z| = 1$.
27. Evaluate $\int_C \frac{(5z-2)\, dz}{z(z-1)}$ where C is the circle $|z| = 2$ described counter clockwise.
28. Evaluate $\int_0^{2\pi} \frac{dz}{2+\cos\theta}$
29. Obtain the Fourier series of $f(x) = x - x^2$ in $(-\pi, \pi)$. Hence deduce $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}$
30. Find the half range cosine series of $f(x) = \begin{cases} \frac{2K}{L}x; & 0 < x < L/2 \\ \frac{2K}{L}(L - x); & L/2 < x < L \end{cases}$
31. Find the Fourier transform of $e^{-x^2/2}$. What is your inference?

Section-IV
Answer any 2 questions from among the questions 32 to 35.
These questions carry 15 marks each.

32. (i) Find an analytic function $f(z) = u + iv$ whose real part is $e^x(x\cos y - y\sin y)$.
    (ii) If $u - v = (x - y)(x^2 + 4xy + y^2)$, find an analytic function $f(z) = u + iv$ in terms of z.
33. Expand $f(z) = \frac{1}{(z-1)\,(z-2)}$ in a series of powers of z valid in:
    (i) $0 < |z| < 1$ (i) $1 < |z| < 2$ (iii) $|z| > 2$
34. Find the Fourier series of $f(x) = \frac{x^2}{2}; -\pi < x < \pi$. Hence deduce: $\sum_{n=1}^{\infty} \frac{1}{n^2}$
35. (i) Show that the Fourier transform is a linear operator
    (ii) Find the Fourier transform of $f(x)$, where

$$f(x) = \begin{cases} e^x; & x < 0 \\ e^{-x}; & x > 0 \end{cases}$$
